

Goniometria

Applicazione di alcune formule goniometriche

Ricordiamo che sussistono le seguenti formule

$$\operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha \cos \beta + \cos \alpha \operatorname{sen}\beta, \quad \operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha \cos \beta - \cos \alpha \operatorname{sen}\beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \operatorname{sen}\alpha \operatorname{sen}\beta, \quad \cos(\alpha - \beta) = \cos \alpha \cos \beta + \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha, \quad \operatorname{sen}\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha, \quad \cos\left(\frac{\pi}{2} - \alpha\right) = \operatorname{sen}\alpha, \quad \cos\left(\frac{\pi}{2} + \alpha\right) = -\operatorname{sen}\alpha;$$

$$\operatorname{sen}(\pi - \alpha) = \operatorname{sen}\alpha, \quad \operatorname{sen}(\pi + \alpha) = -\operatorname{sen}\alpha, \quad \cos(\pi - \alpha) = -\cos \alpha, \quad \cos(\pi + \alpha) = -\cos \alpha;$$

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Es_1

$$\cos(60^\circ + \alpha) + \operatorname{sen}(30^\circ + \alpha) = (\cos 60^\circ \cos \alpha - \operatorname{sen} 60^\circ \operatorname{sen}\alpha) + (\operatorname{sen} 30^\circ \cos \alpha + \cos 30^\circ \operatorname{sen}\alpha) =$$

$$\frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \operatorname{sen}\alpha + \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \operatorname{sen}\alpha = 2 \cdot \frac{1}{2} \cos \alpha = \cos \alpha$$

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Es_2

$$\cos(45^\circ + \alpha) \cos(45^\circ - \alpha) + \operatorname{sen}^2 \alpha = (\cos 45^\circ \cos \alpha - \operatorname{sen} 45^\circ \operatorname{sen}\alpha)(\cos 45^\circ \cos \alpha + \operatorname{sen} 45^\circ \operatorname{sen}\alpha)$$

$$+ \operatorname{sen}^2 \alpha = \left(\frac{\sqrt{2}}{2} \cos \alpha - \frac{\sqrt{2}}{2} \operatorname{sen}\alpha \right) \left(\frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \operatorname{sen}\alpha \right) + \operatorname{sen}^2 \alpha =$$

$$\left(\frac{\sqrt{2}}{2} \right)^2 (\cos \alpha - \operatorname{sen}\alpha)(\cos \alpha + \operatorname{sen}\alpha) + \operatorname{sen}^2 \alpha = \frac{1}{2} (\cos^2 \alpha - \operatorname{sen}^2 \alpha) + \operatorname{sen}^2 \alpha =$$

$$\frac{1}{2} \cos^2 \alpha + \left(-\frac{1}{2} + 1 \right) \operatorname{sen}^2 \alpha = \frac{1}{2} (\cos^2 \alpha + \operatorname{sen}^2 \alpha) = \frac{1}{2}$$

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Es_3

$$-\sqrt{3} \operatorname{sen}\alpha + \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) - 2\sqrt{3} \operatorname{sen}\left(\frac{\pi}{3} - \alpha\right) = -\sqrt{3} \operatorname{sen}\alpha + \cos \alpha - 2\sqrt{3} \left(\operatorname{sen} \frac{\pi}{3} \cos \alpha - \cos \frac{\pi}{3} \operatorname{sen}\alpha \right) =$$

$$-\sqrt{3} \operatorname{sen}\alpha + \cos \alpha - 2\sqrt{3} \left(\frac{\sqrt{3}}{2} \cos \alpha - \frac{1}{2} \operatorname{sen}\alpha \right) = -\sqrt{3} \operatorname{sen}\alpha + \cos \alpha - 3 \cos \alpha + \sqrt{3} \operatorname{sen}\alpha = -2 \cos \alpha$$

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Es_4

$$\frac{\cos(30^\circ + \alpha)}{\cos(30^\circ - \alpha)} - \frac{\operatorname{tg} 30^\circ - \operatorname{tg} \alpha}{\operatorname{tg} 30^\circ + \operatorname{tg} \alpha} = \frac{\cos 30^\circ \cos \alpha - \operatorname{sen} 30^\circ \operatorname{sen} \alpha}{\cos 30^\circ \cos \alpha + \operatorname{sen} 30^\circ \operatorname{sen} \alpha} - \frac{\frac{\sqrt{3}}{3} - \frac{\operatorname{sen} \alpha}{\cos \alpha}}{\frac{\sqrt{3}}{3} + \frac{\operatorname{sen} \alpha}{\cos \alpha}} =$$

$$\frac{\frac{\sqrt{3}}{2} \cos \alpha - \frac{1}{2} \operatorname{sen} \alpha}{\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \operatorname{sen} \alpha} - \frac{\frac{\sqrt{3} \cos \alpha - \operatorname{sen} \alpha}{2}}{\frac{\sqrt{3} \cos \alpha + \operatorname{sen} \alpha}{2}} =$$

Per la semplificazione della seconda frazione è necessaria la condizione $\cos \alpha \neq 0$

$\rightarrow \alpha \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$ e si passa alla seguente espressione:

$$\frac{\sqrt{3} \cos \alpha - \operatorname{sen} \alpha}{\sqrt{3} \cos \alpha + \operatorname{sen} \alpha} - \frac{\sqrt{3} \cos \alpha - 3 \operatorname{sen} \alpha}{\sqrt{3} \cos \alpha + 3 \operatorname{sen} \alpha} =$$

$$\frac{(\sqrt{3} \cos \alpha - \operatorname{sen} \alpha)(\sqrt{3} \cos \alpha + 3 \operatorname{sen} \alpha) - (\sqrt{3} \cos \alpha - 3 \operatorname{sen} \alpha)(\sqrt{3} \cos \alpha + \operatorname{sen} \alpha)}{(\sqrt{3} \cos \alpha + \operatorname{sen} \alpha)(\sqrt{3} \cos \alpha + 3 \operatorname{sen} \alpha)} =$$

$$\frac{(3 \cos^2 \alpha + 2\sqrt{3} \operatorname{sen} \alpha \cos \alpha - 3 \operatorname{sen}^2 \alpha) - (3 \cos^2 \alpha - 2\sqrt{3} \operatorname{sen} \alpha \cos \alpha - 3 \operatorname{sen}^2 \alpha)}{3 \cos^2 \alpha + 4\sqrt{3} \operatorname{sen} \alpha \cos \alpha + 3 \operatorname{sen}^2 \alpha} =$$

$$\frac{4\sqrt{3} \operatorname{sen} \alpha \cos \alpha}{3 + 4\sqrt{3} \operatorname{sen} \alpha \cos \alpha} = \frac{2\sqrt{3} \operatorname{sen}(2\alpha)}{3 + 2\sqrt{3} \operatorname{sen}(2\alpha)} = \frac{2 \operatorname{sen}(2\alpha)}{\sqrt{3} + 2 \operatorname{sen}(2\alpha)}$$

Esercizi proposti

Provare che risulta

- a) $\frac{\operatorname{sen}(30^\circ + \alpha)}{\operatorname{sen}(30^\circ - \alpha)} - \frac{\operatorname{cotg} 30^\circ - \operatorname{cotg} \alpha}{\operatorname{cotg} 30^\circ + \operatorname{cotg} \alpha} = \frac{2(2 - \cos(2\alpha))}{\cos(2\alpha) - 1}, \text{ con } \alpha \neq k\pi, k \in \mathbb{Z}.$
- b) $\frac{\operatorname{sen}(\alpha - 30^\circ)}{\operatorname{sen}(\alpha + 30^\circ)} - \frac{\operatorname{tg} \alpha - \operatorname{tg} 30^\circ}{\operatorname{tg} \alpha + \operatorname{tg} 30^\circ} = \dots = \frac{\sqrt{3} \operatorname{sen} \alpha - \cos \alpha}{\sqrt{3} \operatorname{sen} \alpha + \cos \alpha} - \frac{3 \operatorname{sen} \alpha - \sqrt{3} \cos \alpha}{3 \operatorname{sen} \alpha + \sqrt{3} \cos \alpha} = \dots = 0,$
- con $\alpha \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$