

## Esercitazione sugli integrali impropri

Verificare i risultati indicati per i seguenti integrali impropri

1) 
$$\int_0^1 \frac{1}{\sqrt{1-x}} dx = \lim_{\varepsilon \rightarrow 0^+} \int_0^{1-\varepsilon} \frac{1}{\sqrt{1-x}} dx = 2$$

2) 
$$\int_4^{+\infty} \frac{1}{x^2} dx = \lim_{k \rightarrow +\infty} \int_4^k \frac{1}{x^2} dx = \frac{1}{4}$$

3) 
$$\int_0^{+\infty} \frac{1}{x^2+1} dx = \lim_{k \rightarrow +\infty} \int_0^k \frac{1}{x^2+1} dx = \frac{\pi}{2}$$

4) 
$$\int_0^{+\infty} \frac{1}{4x^2+26} dx = \lim_{k \rightarrow +\infty} \int_0^k \frac{1}{4x^2+26} dx = \frac{\pi\sqrt{26}}{104}$$

5) 
$$\int_0^{+\infty} e^{-x} dx = \lim_{k \rightarrow +\infty} \int_0^k e^{-x} dx = 1$$

6) 
$$\int_2^{+\infty} xe^{-x} dx = \lim_{k \rightarrow +\infty} \int_2^k xe^{-x} dx = 3e^{-2}$$

7) 
$$\int_1^{+\infty} \frac{1}{x+2} dx = \lim_{k \rightarrow +\infty} \int_1^k \frac{1}{x+2} dx = +\infty$$

8) 
$$\int_0^e x \cdot \ln(x) dx = \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^e x \cdot \ln(x) dx = \frac{e^2}{4}$$

9) 
$$\int_1^{+\infty} \frac{\ln(x)}{x^2} dx = \lim_{k \rightarrow +\infty} \int_1^k \frac{\ln(x)}{x^2} dx = 1$$

10) 
$$\int_1^e \frac{1}{x \cdot \ln(x)} dx = \lim_{\varepsilon \rightarrow 0^+} \int_{1+\varepsilon}^e \frac{1}{x \cdot \ln(x)} dx = +\infty$$

11) 
$$\int_0^1 \frac{x}{\sqrt{1-x^2}} dx = \lim_{\varepsilon \rightarrow 0^+} \int_0^{1-\varepsilon} \frac{x}{\sqrt{1-x^2}} dx = 1$$

12) 
$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{\varepsilon \rightarrow 0^+} \int_0^{1-\varepsilon} \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2}$$