

Esercitazione sui limiti della forma $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$

$$1) \lim_{x \rightarrow 1} \frac{3x^2 - 4x + 1}{2x^2 - x - 1} = \lim_{x \rightarrow 1} \frac{(3x-1)(x-1)}{(2x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{3x-1}{2x+1} = \frac{2}{3}$$

$$2) \lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{2 + x - x^2} = \lim_{x \rightarrow -1} \frac{(x+1)^2}{(2-x)(x+1)} = \lim_{x \rightarrow -1} \frac{x+1}{2-x} = 0$$

$$3) \lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{x(x-3)}{(x-3)(x+3)} = \lim_{x \rightarrow 3} \frac{x}{x+3} = \frac{3}{6} = \frac{1}{2}$$

$$4) \lim_{x \rightarrow 1} \frac{(x-2)^2 - (3-2x)^2}{x^3 - 3x^2 + 2x} = \lim_{x \rightarrow 1} \frac{x^2 - 4x + 4 - (9 - 12x + 4x^2)}{x(x^2 - 3x + 2)} = \lim_{x \rightarrow 1} \frac{-3x^2 + 8x - 5}{x(x^2 - 3x + 2)} =$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(5-3x)}{x(x-1)(x-2)} = \lim_{x \rightarrow 1} \frac{5-3x}{x(x-2)} = -2$$

$$5) \lim_{x \rightarrow 1} \frac{\sqrt{5x-1} - 2x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(\sqrt{5x-1} - 2x)(\sqrt{5x-1} + 2x)}{(x^2 - 1)(\sqrt{5x-1} + 2x)} = \lim_{x \rightarrow 1} \frac{(\sqrt{5x-1})^2 - 4x^2}{(x^2 - 1)(\sqrt{5x-1} + 2x)} =$$

$$\lim_{x \rightarrow 1} \frac{-4x^2 + 5x - 1}{(x^2 - 1)(\sqrt{5x-1} + 2x)} = \lim_{x \rightarrow 1} \frac{-4(x-1)\left(x - \frac{1}{4}\right)}{(x-1)(x+1)(\sqrt{5x-1} + 2x)} =$$

$$\lim_{x \rightarrow 1} \frac{-(4x-1)}{(x+1)(\sqrt{5x-1} + 2x)} = -\frac{3}{8}$$

$$6) \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x - \sqrt{x}} = \lim_{x \rightarrow 0} \frac{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)(x + \sqrt{x})}{(x - \sqrt{x})(x + \sqrt{x})(\sqrt{x+1} + 1)} = \lim_{x \rightarrow 0} \frac{(x+1-1)(x + \sqrt{x})}{(x^2 - x)(\sqrt{x+1} + 1)} =$$

$$\lim_{x \rightarrow 0} \frac{x(x + \sqrt{x})}{x(x-1)(\sqrt{x+1} + 1)} = \lim_{x \rightarrow 0} \frac{x + \sqrt{x}}{(x-1)(\sqrt{x+1} + 1)} = \frac{0}{-1 \cdot 2} = 0$$

$$7) \lim_{x \rightarrow +\infty} \frac{x^2 - 3x}{2x^2 - x + 1} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(1 - \frac{3}{x}\right)}{x^2 \left(2 - \frac{1}{x} + \frac{1}{x^2}\right)} = \lim_{x \rightarrow +\infty} \frac{1 - \frac{3}{x}}{2 - \frac{1}{x} + \frac{1}{x^2}} = \frac{1-0}{2-0+0} = \frac{1}{2}$$

$$8) \lim_{x \rightarrow +\infty} \frac{2x - \sqrt{x^2 + x}}{x - 1} = \lim_{x \rightarrow +\infty} \frac{2x - x\sqrt{1 + \frac{1}{x}}}{x - 1} = \lim_{x \rightarrow +\infty} \frac{x \left(2 - \sqrt{1 + \frac{1}{x}}\right)}{x \left(1 - \frac{1}{x}\right)} = \lim_{x \rightarrow +\infty} \frac{2 - \sqrt{1 + \frac{1}{x}}}{1 - \frac{1}{x}} = \frac{2-1}{1} = 1$$

$$9) \lim_{x \rightarrow -\infty} \frac{x + \sqrt{x^2 - x}}{x} = \lim_{x \rightarrow -\infty} \left(\frac{x}{x} + \frac{|x|}{x} \sqrt{1 - \frac{x}{x^2}} \right) = \lim_{x \rightarrow -\infty} \left(1 + \frac{-x}{x} \sqrt{1 - \frac{1}{x}} \right) = \lim_{x \rightarrow -\infty} \left(1 - \sqrt{1 - \frac{1}{x}} \right) =$$

$$1 - 1^+ = 0^-$$

$$10) \lim_{x \rightarrow +\infty} \frac{x\sqrt[3]{x^2} - x + 1}{x\sqrt{x} - \sqrt[3]{x^5}} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{5}{3}} - x + 1}{x^{\frac{3}{2}} - x^{\frac{5}{3}}} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{5}{3}} \left(1 - x^{1-\frac{5}{3}} + x^{-\frac{5}{3}} \right)}{x^{\frac{5}{3}} \left(x^{\frac{3}{2}-\frac{5}{3}} - 1 \right)} = \lim_{x \rightarrow +\infty} \frac{1 - x^{-\frac{2}{3}} + x^{-\frac{5}{3}}}{x^{-\frac{1}{6}} - 1} =$$

$$\frac{1-0+0}{0-1} = -1$$