

Esercitazione sugli integrali indefiniti di funzioni razionali o irrazionali fratte

- 1) $\int \frac{dx}{5x^2 + x + 1} = \frac{2}{\sqrt{19}} \operatorname{arctg}\left(\frac{10x+1}{\sqrt{19}}\right) + c$
- 2) $\int \frac{dx}{9x^2 + 6x + 1} = \int (3x+1)^{-2} dx = \dots = -\frac{1}{3(3x+1)} + c$
- 3) $\int \frac{\pi dx}{x^2 - 2\pi x + \pi^2} = \pi \int (x - \pi)^{-2} dx = \frac{\pi}{\pi - x} + c$
- 4) $\int \frac{2x+2}{x^2 + 2x + 1} dx = 2 \log|x+1| + c$
- 5) $\int \frac{3x-7}{x^2 - 2x + 9} dx = 3 \log|x-3| - \frac{2}{x-3} + c$
- 6) $\int \frac{5x+2}{3x^2 + x + 2} dx = \frac{5}{6} \int \frac{6x+1}{3x^2 + x + 2} dx + \frac{7}{6} \int \frac{1}{3x^2 + x + 2} dx = \frac{5}{6} \log(3x^2 + x + 2) + \frac{7}{\sqrt{23}} \operatorname{arctg}\left(\frac{6x+1}{\sqrt{23}}\right) + c$
- 7) $\int \frac{x+1}{2x^2 - x + 1} dx = \frac{1}{4} \log(2x^2 - x + 1) + \frac{5}{2\sqrt{7}} \operatorname{arctg}\left(\frac{4x-1}{\sqrt{7}}\right) + c$
- 8) $\int \frac{x^2 + 3}{5x^2 + x - 6} dx = \int \frac{1}{5} dx - \frac{1}{5} \int \frac{x-21}{5x^2 + x - 6} dx = \frac{1}{5} x - \frac{111}{275} \log|5x+6| + \frac{4}{11} \log|x-1| + c$
- 9) $\int \frac{x}{x^2 + 10x + 25} dx = \log|x+5| + \frac{5}{x+5} + c$
- 10) $\int \frac{x^3 + 6}{x^2 + x + 2} dx = \frac{x^2}{2} - x - \frac{1}{2} \log(x^2 + x + 2) + \frac{17}{\sqrt{7}} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{7}}\right) + c$
- 11) $\int \frac{dx}{1+4x^2} = \frac{1}{2} \int \frac{2dx}{1+(2x)^2} = \frac{1}{2} \operatorname{arctg}(2x) + c$
- 12) $\int \frac{dx}{2+10x^2} = \frac{1}{2\sqrt{5}} \operatorname{arctg}(\sqrt{5}x) + c$
- 13) $\int \frac{3dx}{4+8x^2} = \frac{3}{4\sqrt{2}} \operatorname{arctg}(\sqrt{2}x) + c$
- 14) $\int \frac{2dx}{1+5x^2} = \frac{2}{\sqrt{5}} \operatorname{arctg}(\sqrt{5}x) + c$
- 15) $\int \frac{dx}{1+(7x-8)^2} = \frac{1}{7} \operatorname{arctg}(7x-8) + c$
- 16) $\int \frac{(2x+3)dx}{1+(x^2+3x)^2} = \operatorname{arctg}(x^2+3x) + c$
- 17) $\int \frac{x-6}{16x^2+1} dx = \frac{1}{32} \log(16x^2+1) - \frac{3}{2} \operatorname{arctg}(4x) + c$
- 18) $\int \frac{3x+20}{8x^2+1} dx = \frac{3}{16} \log(1+8x^2) + \frac{10}{\sqrt{2}} \operatorname{arctg}(2\sqrt{2}x) + c$

$$19) \int \frac{dx}{\sqrt{1-3x^2}} = \frac{1}{\sqrt{3}} \arcsen(\sqrt{3}x) + c$$

$$20) \int \frac{x+2}{\sqrt{1-4x^2}} dx = \int x(1-4x^2)^{-\frac{1}{2}} dx + \int \frac{2}{\sqrt{1-(2x)^2}} dx = -\frac{1}{8} \sqrt{1-4x^2} + \arcsen(2x) + c$$

$$21) \int \frac{8x-1}{1+(4x^2-x)^2} dx = \arctg(4x^2-x) + c$$