

Esercizi sugli integrali indefiniti

Premessa

Per la risoluzione degli integrali che seguono si tengano presenti gli integrali immediati

$$1) \int dx = x + c, \forall c \in R$$

$$2) \int k dx = kx + c, \forall k \in R \text{ e } \forall c \in R$$

$$3) \int x dx = \frac{x^2}{2} + c, \forall c \in R$$

$$4) \int x^n dx = \frac{x^{n+1}}{n+1} + c, \forall n \in R, n \neq -1 \text{ e } \forall c \in R$$

$$5) \int x^{-1} dx = \int \frac{1}{x} dx = \log|x| + c, \forall c \in R$$

$$6) \int \frac{\varphi'(x)}{\varphi(x)} dx = \log|\varphi(x)| + c, \forall c \in R$$

$$7) \int \frac{k}{mx+n} dx = \frac{k}{m} \int \frac{m}{mx+n} dx = \frac{k}{m} \log|mx+n| + c, \text{ con } m \neq 0 \text{ e } \forall c \in R$$

$$8) \int \frac{P(x)}{mx+n} dx = \int \left(Q(x) + \frac{k}{mx+n} \right) dx = \int Q(x) dx + \int \frac{k}{mx+n} dx, \text{ essendo } P(x) \text{ un polinomio}$$

intero di grado $n \geq 1$, $m \neq 0$, $Q(x)$ il polinomio quoziente nella divisione $P(x):(mx+n)$, k il resto nella stessa divisione.

e le regole di integrazione

$$1) \int k \cdot f(x) dx = k \cdot \int f(x) dx, \forall k \in R$$

$$2) \int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

Esercizi

$$1) \int (3x+1) dx = 3 \int x dx + \int 1 dx = \frac{3}{2} x^2 + x + c$$

$$2) \int \left(x^3 - 2x^2 - \frac{1}{2} x \right) dx = \int x^3 dx - 2 \int x^2 dx - \frac{1}{2} \int x dx = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{4} + c$$

$$3) \int (6x^3 - 3x^2 - 4x - 1) dx = 6 \int x^3 dx - 3 \int x^2 dx - 4 \int x dx - \int 1 dx = 6 \cdot \frac{x^4}{4} - 3 \cdot \frac{x^3}{3} - 4 \cdot \frac{x^2}{2} - x + c =$$
$$\frac{3x^4}{2} - x^3 - 2x^2 - x + c$$

$$4) \int (x^2 - 7x)(-x+5) dx = \dots = -\frac{x^4}{4} + 4x^3 - \frac{35}{2} x^2 + c$$

$$5) \int (x\sqrt{x} + 2) dx = \int \left(xx^{\frac{1}{2}} + 2 \right) dx = \int \left(x^{\frac{3}{2}} + 2 \right) dx = \int x^{\frac{3}{2}} dx + 2 \int dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 2x + c =$$

$$\frac{2}{5} x^{\frac{5}{2}} + 2x + c$$

$$6) \int (\sqrt[4]{x^3} - x\sqrt[3]{x}) dx = \int \left(x^{\frac{3}{4}} - xx^{\frac{1}{3}} \right) dx = \int x^{\frac{3}{4}} dx - \int x^{\frac{4}{3}} dx = \frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} - \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + c = \frac{4}{7} x^{\frac{7}{4}} - \frac{3}{7} x^{\frac{7}{3}} + c$$

$$7) \int (x^2\sqrt{x} - 3\sqrt[3]{x^2}) dx = \dots = \frac{2}{7} x^{\frac{7}{2}} - \frac{9}{5} x^{\frac{5}{3}} + c$$

$$8) \int \left(\frac{2x^5 - 6x^2 + 5x - 2}{x^4} \right) dx = \int \left(2x - \frac{6}{x^2} + \frac{5}{x^3} - \frac{2}{x^4} \right) dx =$$

$$2 \int x dx - 6 \int x^{-2} dx + 5 \int x^{-3} dx - 2 \int x^{-4} dx = 2 \cdot \frac{x^2}{2} - 6 \cdot \frac{x^{-2+1}}{-2+1} + 5 \cdot \frac{x^{-3+1}}{-3+1} - 2 \cdot \frac{x^{-4+1}}{-4+1} + c =$$

$$x^2 + \frac{6}{x} - \frac{5}{2x^2} + \frac{2}{3x^3} + c$$

$$9) \int \left(\frac{2x^5 - 3x^2 + 7x - 1}{x^5} \right) dx = \dots = 2x + \frac{3}{2x^2} - \frac{7}{3x^3} + \frac{1}{4x^4} + c$$

$$10) \int \left(\frac{x^4 + 2x^2 - 5x}{x^5} \right) dx = \dots = \log|x| - \frac{1}{x^2} + \frac{5}{3x^3} + c$$

$$11) \int \left(\frac{2x^2 + 3x - 1}{x} \right) dx = \int \left(2x + 3 - \frac{1}{x} \right) dx = 2 \int x dx + 3 \int dx - \int \frac{1}{x} dx = x^2 + 3x - \log|x| + c$$

$$12) \int \left(\frac{4x^2 - 3x + 7}{-x} \right) dx = \dots = -2x^2 + 3x - 7 \log|x| + c$$

$$13) \int \left(5x - \frac{1}{x} + \frac{3x^2 - 8x}{x^3} \right) dx = \int \left(5x - \frac{1}{x} + \frac{3}{x} - \frac{8}{x^2} \right) dx = \int \left(5x + \frac{2}{x} - \frac{8}{x^2} \right) dx =$$

$$5 \int x dx + 2 \int \frac{1}{x} dx - 8 \int x^{-2} dx = \frac{5}{2} x^2 + 2 \log|x| + \frac{8}{x} + c$$

$$14) \int \left(2x - \frac{1}{x} + \frac{3x^2 - 5x}{x^3} \right) dx = \dots = x^2 + 2 \log|x| + \frac{5}{x} + c$$

$$15) \int \frac{4}{5x+6} dx = \frac{4}{5} \int \frac{5}{5x+6} dx = \frac{4}{5} \log|5x+6| + c$$

$$16) \int \frac{16}{2x-15} dx = 8 \int \frac{2}{2x-15} dx = 8 \log|2x-15| + c$$

$$17) \int \frac{-6}{3x-1} dx = -2 \int \frac{3}{3x-1} dx = -2 \log|3x-1| + c$$

$$18) \int \frac{4}{17-x} dx = \dots = -4 \log|17-x| + c$$

- 19) $\int \frac{\sqrt{2}}{2+\sqrt{10x}} dx = \frac{1}{\sqrt{5}} \int \frac{\sqrt{5} \cdot \sqrt{2}}{2+\sqrt{10x}} dx = \frac{1}{\sqrt{5}} \log|2+\sqrt{10x}| + c$
- 20) $\int \left(\frac{-2}{4x+1} + \frac{5}{10x-2} \right) dx = \dots = -\frac{1}{2} \log|4x+1| + \frac{5}{2} \log|10x-2| + c$
- 21) $\int \frac{6x-3}{2x+3} dx = \int \left(3 - \frac{12}{2x+3} \right) dx = 3x - 6 \log|2x+3| + c$
- 22) $\int \frac{4x-7}{2x-3} dx = \int \left(2 - \frac{1}{2x-3} \right) dx = 2x - \frac{1}{2} \log|2x-3| + c$
- 23) $\int \frac{4x^3 - x + 2}{2x-1} dx = \int \left(2x^2 + x + \frac{2}{2x-1} \right) dx = \frac{2}{3} x^3 + \frac{x^2}{2} + \log|2x-1| + c$
- 24) $\int \frac{x^3 - 2x^2 - 6}{x-5} dx = \int \left(x^2 + 3x + 15 + \frac{69}{x-5} \right) dx = \frac{2}{3} x^3 + \frac{3}{2} x^2 + 15x + 69 \log|x-5| + c$
- 25) $\int \frac{x + \frac{1}{x}}{1 - \frac{1}{x}} dx = (\text{con } x \neq 0) \int \frac{x^2 + 1}{x-1} dx = \int \left(x + 1 + \frac{2}{x-1} \right) dx = \frac{1}{2} x^2 + x + 2 \log|x-1| + c$

Altri esercizi- Il lettore verifichi che risulta

- 26) $\int \left(x^2 - \frac{1}{x} \right) \left(x + \frac{2}{x^3} \right) dx = \dots = \frac{1}{4} x^4 + 2 \log|x| - x + \frac{2}{3x^3} + c$
- 27) $\int (3x-1)^2 \left(x - \frac{1}{x^3} \right) dx = \dots = \frac{9}{4} x^4 - 2x^3 + \frac{1}{2} x^2 - 9 \log|x| - \frac{6}{x} + \frac{1}{2x^2} + c$
- 28) $\int \left(-2x^2 \sqrt[3]{x^2} + \frac{1}{3} \sqrt[4]{x^3} \right) dx = \dots = -\frac{6}{11} x^{\frac{11}{3}} + \frac{4}{21} x^{\frac{7}{4}} + c$
- 29) $\int \left(\frac{4x^3 - 5x^2 - 6x - 1}{12x^4} \right) dx = \dots = \frac{1}{3} \log|x| + \frac{5}{12x} + \frac{1}{4x^2} + \frac{1}{36x^3} + c$
- 30) $\int \frac{10}{12x+3} dx = \dots = \frac{5}{6} \log|12x+3| + c$
- 31) $\int \frac{4x-7}{2x+3} dx = \dots = 2x - \frac{13}{2} \log|2x+3| + c$
- 32) $\int \frac{6x+11}{3x-8} dx = \dots = 2x + 9 \log|3x-8| + c$
- 33) $\int \frac{9x-10}{3x+4} dx = \dots = 3x - \frac{22}{3} \log|3x+4| + c$
- 34) $\int \frac{x^3+1}{x-1} dx = \dots = \frac{1}{3} x^3 + \frac{1}{2} x^2 + x + 2 \log|x-1| + c$
- 35) $\int \left(\left(x^2 - \frac{1}{2} \right) : \left(x - \frac{1}{2} \right) \right) dx = \dots = \frac{1}{2} x^2 + \frac{1}{2} x - \frac{1}{4} \log \left| x - \frac{1}{2} \right| + c$