Esercizi sul calcolo della derivata prima

1) $y=(4x-5)^{10}(5x+4)^{8}$

$$y'=10(4x-5)^{9}\cdot 4\cdot(5x+4)^{8}+(4x-5)^{10}\cdot 8\cdot(5x+4)^{7}\cdot 4=\ldots=40(4x-5)^{9}(5x+4)^{7}(9x-1)$$

2) $y=\dfrac{(2x-3)^{3}}{(3x+2)^{2}}$

$$y'=\frac{3(2x-3)^{2}\cdot 2\cdot(3x+2)^{2}-(2x-3)^{3}\cdot 2(3x+2)\cdot 3}{(3x+2)^{4}}=\ldots=\frac{6(2x-3)^{2}(x+5)}{(3x+2)^{3}}$$

3) $y=\dfrac{x^{2}+1}{x^{3}+1}$

$$y'=\frac{2x(x^{3}+1)-(x^{2}+1)\cdot 3x^{2}}{(x^{3}+1)^{2}}=\ldots=\frac{x(2-3x-x^{3})}{(x^{3}+1)^{2}}$$

4) $y=\dfrac{x^{2}-1}{x^{3}+1}$, si noti che $\forall x\neq -1$ risulta $y=\dfrac{x^{2}-1}{x^{3}+1}=\dfrac{x-1}{x^{2}-x+1}$

$$y'=\frac{2x-x^{2}}{(x^{2}-x+1)^{2}}$$

5) $y=\dfrac{x^{-2}+x^{-1}}{x^{2}+x+1}$, $\qquad y'=-\dfrac{3x^{3}+6x^{2}+4x+2}{x^{3}(x^{2}+x+1)^{2}}$

6) $y=\log\left(x+\sqrt{x^{2}+1}\right)$

$$y'=\frac{1}{x+\sqrt{x^{2}+1}}\cdot\left(1+\frac{2x}{2\sqrt{x^{2}+1}}\right)=\frac{1}{x+\sqrt{x^{2}+1}}\cdot\left(\frac{\sqrt{x^{2}+1}+x}{\sqrt{x^{2}+1}}\right)=\frac{1}{\sqrt{x^{2}+1}}$$

7) $y=arcsen\left(e^{x}-1\right)$

$$y'=\frac{e^{x}}{\sqrt{1-(e^{x}-1)^{2}}}=\ldots=\frac{e^{\frac{x}{2}}}{\sqrt{2-e^{x}}}$$

8) $y=arcsen\left(\dfrac{1}{x}\right)$

$$y'=\frac{1}{\sqrt{1-\frac{1}{x^2}}}\left(-\frac{1}{x^2}\right)=\frac{|x|}{\sqrt{x^2-1}}\left(-\frac{1}{x^2}\right)=-\frac{1}{|x|\sqrt{x^2-1}}$$

9) $y=\arccos(4x+1)$

$$y'=-\frac{1}{\sqrt{1-(4x+1)^2}}\cdot 4=..=-\frac{2}{\sqrt{-2x-4x^2}}$$

10) $y=\arccos\left(4^x+1\right)$

Si osservi che la funzione non è definita per alcun valore reale di x, quindi non ha senso calcolare la funzione derivata prima.

11) $y=\frac{4^x+1}{2^x+2}$

$$y'=\frac{4^x\cdot\log 4\cdot\left(2^x+2\right)-\left(4^x+1\right)\cdot 2^x\cdot\log 2}{\left(2^x+2\right)^2}=...=\frac{2^x\left(2^{2x}+4\cdot 2^x-1\right)\cdot\log 2}{\left(2^x+2\right)^2}$$

12) $y=arctg\left(\frac{1}{\sqrt{x^2-1}}\right)+\sqrt{x^2-1}$, la funzione è definita in $]-\infty;-1[\cup]1;+\infty[$

$$y'=\frac{1}{1+\frac{1}{x^2-1}}\cdot D\left(\left(x^2-1\right)^{-\frac{1}{2}}\right)+\frac{2x}{2\sqrt{x^2-1}}=\frac{x^2-1}{x^2}\cdot\left(-\frac{1}{2}\right)\left(x^2-1\right)^{-\frac{3}{2}}\cdot 2x+\frac{x}{\sqrt{x^2-1}}=$$

$$-\frac{\left(x^2-1\right)^{-\frac{1}{2}}}{x}+\frac{x}{\sqrt{x^2-1}}=-\frac{1}{x\sqrt{x^2-1}}+\frac{x}{\sqrt{x^2-1}}=\frac{x^2-1}{x\sqrt{x^2-1}}=\frac{\sqrt{x^2-1}}{x}$$

13) $y=\sqrt{\frac{sen^2x+tgx}{sen(2x)}}=\sqrt{\frac{sen^2x+\frac{senx}{\cos x}}{2senx\cos x}}=\sqrt{\frac{\cancel{senx}\left(senx+\frac{1}{\cos x}\right)}{2\,\cancel{senx}\cos x}}=(\text{con } x\neq k\pi)$

$\sqrt{\left(senx+\frac{1}{\cos x}\right)\frac{1}{2\cos x}}=\sqrt{\left(\frac{1}{2}sen(2x)+1\right)\frac{1}{2\cos^2 x}}=\frac{1}{2}\sqrt{\frac{sen(2x)+2}{\cos^2 x}}$, la derivata prima è

$$y'=\frac{1}{2}\cdot\frac{1}{2\sqrt{\frac{sen(2x)+2}{\cos^2 x}}}\cdot\frac{2\cos(2x)\cdot\cos^2 x-\left(sen(2x)+2\right)\cdot 2\cos x(-senx)}{\cos^4 x}=$$

$$\frac{|\cos x|}{4\sqrt{sen(2x)+2}}\cdot\frac{2\cos x}{\cos^4 x}\cdot\left(\cos(2x)\cdot\cos x+sen(2x)\cdot senx+2senx\right)=\frac{|\cos x|}{2\sqrt{sen(2x)+2}}\cdot\frac{1}{\cos^3 x}\cdot$$

$$\left(\left(2\cos^2 x-1\right)\cdot\cos x+2sen^2x\cdot\cos x+2senx\right)=\frac{|\cos x|}{2\sqrt{sen(2x)+2}}\cdot\frac{1}{\cos^3 x}\cdot$$

$$\left(2\cos^3 x-\cos x+2\left(1-\cos^2 x\right)\cdot\cos x+2senx\right)=\frac{\cos x+2senx}{2|\cos x|\cdot\cos x\sqrt{sen(2x)+2}}$$

14) $y=\arccos(senx)$

$$y'=-\frac{1}{\sqrt{1-sen^2x}}\cdot\cos x=-\frac{\cos x}{\sqrt{\cos^2 x}}=-\frac{\cos x}{|\cos x|}$$

Possiamo esplicitare la funzione derivata prima come segue

$$y'=\begin{cases}-1 \quad per \quad \left(-\frac{\pi}{2}+2k\pi<x<\frac{\pi}{2}+2k\pi, k\in Z\right)\\ 1 \quad per \quad \left(\frac{\pi}{2}+2k\pi<x<\frac{3\pi}{2}+2k\pi, k\in Z\right)\end{cases}$$

15) $y=arctg\left(1-\frac{1}{x}\right)$

$$y'=\frac{1}{1+\left(1-\frac{1}{x}\right)^2}\cdot\left(\frac{1}{x^2}\right)=\frac{x^2}{x^2+(x-1)^2}\cdot\left(\frac{1}{x^2}\right)=\frac{1}{x^2+(x-1)^2},\forall x\neq 0$$

16) $y=arctg\left(e^x-e^{-x}\right)$

$$y'=\frac{e^x+e^{-x}}{1+\left(e^x-e^{-x}\right)^2}=\frac{e^{-x}\left(e^{2x}+1\right)}{e^{2x}+e^{-2x}-1}=\frac{\left(e^{2x}+1\right)}{e^x\left(e^{2x}+e^{-2x}-1\right)}=\frac{e^{2x}+1}{e^{3x}+e^{-x}-e^x}$$

17) $y=tgx\cdot\log(tgx)$

$$y'=\left(1+tg^2x\right)\cdot\log(tgx)+tgx\cdot\frac{1}{tgx}\cdot\left(1+tg^2x\right)=\left(1+tg^2x\right)\left(\log(tgx)+1\right)$$

18) $y=\log_x^{(1-x)}=\frac{\log(1-x)}{\log x}$

$$y'=\frac{\frac{-1}{1-x}\cdot\log x-\log(1-x)\cdot\frac{1}{x}}{\log^2 x}=\ldots=\frac{x\log x-(x-1)\log(x-1)}{x(x-1)\log^2 x}$$

19) $y = x^{\sqrt{x}}$, $y' = x^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\cdot\log x + \sqrt{x}\cdot\frac{1}{x}\right) = \frac{x^{\sqrt{x}}}{2\sqrt{x}}\left(2+\log x\right)$

20) $y = \left(1-\cos x\right)^{senx}$

$$y' = \left(1-\cos x\right)^{senx}\cdot\left(\cos x\cdot\log\left(1-\cos x\right) + senx\cdot\frac{senx}{1-\cos x}\right) = \left(1-\cos x\right)^{senx}\cdot$$
$$\left(\cos x\cdot\log\left(1-\cos x\right) + \frac{\left(1-\cos x\right)\left(1+\cos x\right)}{1-\cos x}\right) =$$
$$\left(1-\cos x\right)^{senx}\cdot\left(\cos x\cdot\log\left(1-\cos x\right) + 1 + \cos x\right)$$