

Esercitazione sulle equazioni di secondo grado in R

Equazioni pure

$$1) \quad 2x^2 - 8 = 0 \rightarrow x^2 = \frac{8}{2} = 4 \rightarrow x = \pm\sqrt{4} = \pm 2 \quad S = \{-2; 2\}$$

$$2) \quad x^2 + 6 = 0 \rightarrow x^2 = -6 \quad \text{L'equazione non ha radici reali} \quad S = \emptyset$$

$$3) \quad (x-3)^2 - 6x = 10 - 12x \rightarrow x^2 - 6x + 9 - 6x = 10 - 12x \rightarrow x^2 = 1 \rightarrow x = \pm 1 \quad S = \{-1; 1\}$$

$$4) \quad (x-1)^2 - 2x + 6 = (2x-1)^2 \rightarrow x^2 - 2x + 1 - 2x + 6 = 4x^2 - 4x + 1 \rightarrow 3x^2 = 6 \rightarrow$$

$$x^2 = 2 \rightarrow x = \pm\sqrt{2} \quad S = \{-\sqrt{2}; \sqrt{2}\}$$

$$5) \quad (3x+2)^2 - 12 = 12x \rightarrow 9x^2 = 8 \rightarrow x = \pm\sqrt{\frac{8}{9}} = \pm\frac{2}{3}\sqrt{2} \quad S = \left\{-\frac{2}{3}\sqrt{2}; \frac{2}{3}\sqrt{2}\right\}$$

$$6) \quad (x-1)^2 - 2(x+1)^2 + 6x = 7 \rightarrow x^2 = -8 \quad \text{L'equazione non ha radici reali.} \quad S = \emptyset$$

Equazioni spurie

$$7) \quad 5x^2 - 7x = 0 \rightarrow x(5x-7) = 0 \Leftrightarrow (x=0) \vee (5x-7=0), \text{ le due radici dell'equazione sono}$$

$$x_1 = 0, x_2 = \frac{7}{5}. \quad S = \left\{0; \frac{7}{5}\right\}$$

$$8) \quad x(2x-1) = (5x-1)^2 - 1 \rightarrow -23x^2 + 9x = 0 \rightarrow x(23x-9) = 0 \Leftrightarrow (x=0) \vee (23x-9=0) \rightarrow$$

$$x_1 = 0, x_2 = \frac{9}{23} \quad S = \left\{0; \frac{9}{23}\right\}$$

$$9) \quad (3-4x)(3+4x) - x(5x-6) = 9 \rightarrow 9 - 16x^2 - 5x^2 + 6x - 9 = 0 \rightarrow 3x(7x-2) = 0 \Leftrightarrow$$

$$(x=0) \vee (7x-2=0) \rightarrow x_1 = 0, x_2 = \frac{2}{7} \quad S = \left\{0; \frac{2}{7}\right\}$$

$$10) \quad x - \frac{1}{2}(x-2)^2 = \frac{1}{2}(x-4) \rightarrow x - \frac{1}{2}(x^2 - 4x + 4) = \frac{1}{2}x - 2 \rightarrow -\frac{1}{2}x^2 + \frac{5}{2}x = 0 \rightarrow$$

$$x(x-5) = 0 \rightarrow x_1 = 0, x_2 = 5 \quad S = \{0; 5\}$$

$$11) \quad 3\left(x - \frac{1}{2}\right)^2 - 4x^2 = \frac{3}{4} - 5x \rightarrow 3\left(x^2 - x + \frac{1}{4}\right) - 4x^2 = \frac{3}{4} - 5x \rightarrow -x^2 + 2x = 0 \rightarrow x_1 = 0, x_2 = 2$$

$$S = \{0; 2\}$$

$$12) \quad (3x-2)(3x+2) = x(4x-5) - 4 \rightarrow 9x^2 - 4 = 4x^2 - 5x - 4 \rightarrow 5x^2 + 5x = 0 \rightarrow x(x+1) = 0 \rightarrow$$

$$x_1 = 0, x_2 = -1 \quad S = \{-1; 0\}$$

Equazioni complete

$$13) 2x^2 + x - 21 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+168}}{4} = \frac{-1 \pm 13}{4} \rightarrow x_1 = -\frac{14}{4} = -\frac{7}{2}; x_2 = 3$$

$$x_1 = -\frac{14}{4} = -\frac{7}{2}; x_2 = 3$$

$$S = \left\{ -\frac{7}{2}; 3 \right\}$$

$$14) (3x-1)^2 + 5x(x-1) = 4 \rightarrow 14x^2 - 11x - 3 = 0 \rightarrow x = \frac{11 \pm \sqrt{289}}{28} = \frac{11 \pm 17}{28} \rightarrow$$

$$x_1 = -\frac{3}{14}; x_2 = 1$$

$$S = \left\{ -\frac{3}{14}; 1 \right\}$$

$$15) 2x^2 + x - 10 = 0 \rightarrow x = \frac{-1 \pm \sqrt{81}}{4} = \frac{-1 \pm 9}{4} \rightarrow x_1 = 2; x_2 = -\frac{10}{4} = -\frac{5}{2}$$

$$S = \left\{ -\frac{5}{2}; 2 \right\}$$

$$16) (x-1)^2 + 7(x-1) - 8 = 0 \rightarrow x^2 + 5x - 14 = 0 \rightarrow x = \frac{-5 \pm \sqrt{81}}{2} = \frac{-5 \pm 9}{2} \rightarrow$$

$$x_1 = -7; x_2 = 2$$

$$S = \{-7; 2\}$$

$$17) x^2 - 5x - (2x-3)^2 = -5 \rightarrow 3x^2 - 7x + 4 = 0 \rightarrow x = \frac{7 \pm 1}{6} \rightarrow x_1 = 1; x_2 = \frac{4}{3}$$

$$S = \left\{ 1; \frac{4}{3} \right\}$$

$$18) 8(x-1)^3 - 7x = (2x-1)^3 - 8 \rightarrow 8(x^3 - 3x^2 + 3x - 1) - 7x = (8x^3 - 12x^2 + 6x - 1) - 8 \rightarrow$$

$$(8-8)x^3 + (-24+12)x^2 + (24-7-6)x + 1 = 0 \rightarrow -12x^2 + 11x + 1 = 0 \rightarrow$$

$$12x^2 - 11x - 1 = 0 \rightarrow x = \frac{11 \pm \sqrt{121+48}}{24} = \frac{11 \pm 13}{24} \rightarrow x_1 = -\frac{1}{12}; x_2 = 1$$

$$S = \left\{ -\frac{1}{12}; 1 \right\}$$

$$19) (x-2)^3 - (x+1)^2(x-1) + 9(x-1) = 0 \rightarrow x^3 - 6x^2 + 12x - 8 - (x+1)(x^2 - 1) + 9x - 9 = 0 \rightarrow$$

$$x^3 - 6x^2 + 12x - 8 - x^3 + x - x^2 + 1 + 9x - 9 = 0 \rightarrow 7x^2 - 22x + 16 = 0 \rightarrow x_1 = \frac{8}{7}; x_2 = 2$$

$$S = \left\{ \frac{8}{7}; 2 \right\}$$

$$20) (x-1)^3 - x(x+1)^2 + 4 = 0 \rightarrow x^3 - 3x^2 + 3x - 1 - x(x^2 + 2x + 1) + 4 = 0 \rightarrow 5x^2 - 2x - 3 = 0 \rightarrow$$

$$x = \frac{1 \pm 4}{5} \rightarrow x_1 = -\frac{3}{5}; x_2 = 1$$

$$S = \left\{ -\frac{3}{5}; 1 \right\}$$

$$21) 9(x-3)^3 - x(3x-1)^2 = -12(5x+1) \rightarrow$$

$$9(x^3 - 9x^2 + 27x - 27) - x(9x^2 - 6x + 1) = -60x - 12 \rightarrow -75x^2 + 302x - 231 = 0 \rightarrow$$

$$75x^2 - 302x + 231 = 0 \rightarrow \frac{\Delta}{4} = 151^2 - 75 \cdot 231 = 5476 = 74^2$$

$$x = \frac{151 \pm \sqrt{74^2}}{75} = \frac{151 \pm 74}{75} \rightarrow x_1 = \frac{77}{75}; x_2 = 3$$

$$S = \left\{ \frac{77}{75}; 3 \right\}$$